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بناء الرموز الخطية لتقاطعات المستويات باستخدام نقاط الإطار في PG(3,2)

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المستخلص:

يهدف هذا البحث إلى بناء رموز خطية لتقاطع المستويات في الفضاء الإسقاطي PG(3,2) الذي يحتوي على خمسة عشر مستوى. من خلال تقاطع هذه المستويات مع بعضها البعض نحصل على ١٠٥ خطا إسقاطيا يحتوي كل منها على ثلاث نقاط يرمز S_i من خلال اتحاد كل S_i مع نقاط الإطار

$F = \{[1,0,0,0], [0,1,0,0], [0,0,1,0], [0,0,0,1], [1,1,1,1]\}$ نحصل على مستويات جديدة، يرمز إليها بالرمز C_i، حيث $i = 1,2, \dots ,105$. تمثل النقاط الناتجة في صورة مصفوفة التوليد ، تم بناء ١٠٥ رمزا خطيا ، اثبت ان الرمز ٢٩ له المعلمات $[6,4,2]_2 - code$ و الرمز ٣٧ له المعلمات $[7,4,2]_2 - code$ و ١٨ $[8,4,4]_2 - code$ و ١٢ له $[8,4,2]_2 - code$ و ٩ له $[7,4,3]_2 - code$ حققت تطبيق (Griesmer bound) ان جميع الرموز ال ١٠٥ هي رموز خطية مثالية .

لكلمات المفتاحية: الفضاء الإسقاطي؛ PG(3,2)؛ الإطار الإسقاطي؛ الشفرات الخطية الثنائية؛ مصفوفة التوليد؛ حد غريسمر.

Construction the linear codes for planes intersection using the frame points in PG (3,2)

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Abstract:

The aim of the research is to Constructing linear codes 〖[n,x,y]〗_h-code (where n the length and x dimension y distance) for the intersection of the planes in the projective space PG(3,2) where it contains fifteen planes .by the intersection of the planes with each other, we obtain 105 projective lines each containing three points which we denote by the symbol S_i where i=1,…,105 ,by taking the union of each S_i with the frame points F={[1,0,0,0],[0,1,0,0],[0,0,1,0],[0,0,0,1],[1,1,1,1]} we obtain new planes, denoted by the symbol C_i , where i= 1,2,…,105. Using the resulting points to form a generator matrix,105 linear codes were constructed Show that 29 has parameters 〖[6,4,2]〗_2-code, 37 has parameters 〖[7,4,2]〗_2-code, 18 has 〖[8,4,4]〗_2-code, 12 has 〖[8,4,2]〗_2-code, and 9 has 〖[7,4,3]〗_2-code. Rigorous application of the Griesmer bound demonstrated that all 105 of these codes are optimal linear codes.

Keywords: Projective space; PG(3,2); projective frame; binary linear codes; generator matrix; Griesmer bound

Introduction

Gökpınar (2010) mentioned that A simulation study on decoding of binary linear codes using projective geometry PG(3,2).Hirschfeld (1998) mentioned that planar projective geometry over finite fields has attracted the attention of many scientists. Al-Seraji (2012) also noted several works that establish relationships between projective geometry and coding theory. Two papers were very important in establishing the connection between the projective plane of order 17 and error correction codes. In one of these papers, that of Al-Seraji (2012), the focus was on optimal codes. AL-Zangana (2012) presented a classification of some concepts and tools for notation theorems. The classification of concepts and tools for coding theory was presented by Hill (1986), while Al-Seraji and Easa (2023) were responsible for the construction of linear codes in PG(1,31).

Previous studies have explored the relationship between projective geometry and symbol theory in numerous mathematical problems. Among the most important of these constructs are bounded projective spaces, including the projective space PG(3,2), which consists of a finite set of 15 points, 35 lines, and 15 planes. There are also five points, three of which are not collinear. This set of five points is called a frame and is denoted by Let F = {[1,0,0,0], [0,1,0,0], [0,0,1,0], [0,0,0,1], [1,1,1,1]}. points, lines, and planes connected by specific geometric relationships. PG(3,2) is suitable for this type of study due to its finite and clear structure, allowing for the study of its points, planes, and intersection relationships. The intersection of two different planes in PG(3,2) produces a projective line containing three points. This structure can be used to construct point compositions and represent them using appropriate matrices, thereby building two-dimensional linear symbols and studying their properties. This research aims to construct a set of linear symbols by representing the resulting points as matrices for generating and constructing the linear symbols from them. It also includes studying the resulting symbol parameters, determining their minimum distance, and verifying their optimality based on the Griesmer Bound.

Definition 1(Hill ,1986):

If T_1 and T_2 are sets we define the union $T_1 \cup T_2$ of T_1 and T_2 to be the set of all elements in either T_1 or T_2. We define the intersection $T_1 \cap T_2$ of T_1 and T_2 to be the set of all elements which are members of both T_1 and T_2. thus

$T_1 \cup T_2 = \{a \in T_1 \ or \ a \in T_2\}, T_1 \cap T_2 = \{a \in T_1 \ and \ a \in T_2\}$

Definition 2 (Radhi & AL-Zangana , 2023)**:**

A generator matrix of a linear $[n, x, y]_h - code\ C$ is $x \times n$ matrix over finite field, it is denoted by G.

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Theorem 3 (AL-Zangana & Shehab , 2021)**:**
A projective 3-space PG(3,h) satisfies the following:
A) Every line contains exactly $h+1$ points and every point is on exactly $h+1$ lines.
B) Every plane contains exactly h^2+h+1 points (lines) and every point is on exactly h^2+h+1 planes.
C) There exist $h^3+h^2+h+1$of points and there exists h^3+h^2+h+1 of planes.
D) Any two planes intersect in exactly $h+1$ points and any line is on exactly $h+1$ planes, and any two points are on exactly $h+1$ planes.

Theorem 4 (Yahya, 2014) **:**The points of PG(3,h) have unique forms which are [1,0,0,0], [v, 1,0,0], [v, w, 1,0], [v, w, z, 1], for all v, w, z in GF(h).

Theorem 5 (Khalaf & Yahya , 2023) **:**
Let C be a $[n,x,y]_q$ code. Then $n \geq g_q(x,y)=\sum_{i=0}^{x-1}\left\lceil\frac{y}{h^i}\right\rceil$. .A Linear codes attaining the Griesmer bound, linear codes with parameters $[g_q(x,y),x,y]_q$, are called Griesmer codes.

1. The PG (3, 2).

In PG (3, 2), the projective space of three dimensions and order 2, has 15 points, 35 lines and 15 planes, the points in PG (3, 2):
m_1=[1,0,0,0],m_2=[0,1,0,0],m_3=[0,0,1,0],m_4=[0,0,0,1],m_5=[1,1,0,0],m_6=[0,1,1,0],
m_7=[0,0,1,1],m_8=[1,1,0,1],m_9=[1,0,1,0],m_{10}=[0,1,0,1],,m_{11}=[1,1,1,0],
m_{12}=[0,1,1,1], m_{13}=[1,1,1,1], m_{14}=[1,0,1,1], m_{15}=[1,0,0,1] }, the five points such that no four of them on a line (collinear) which called the frame and its denoted by F = {m_1 , m_2 , m_3 , m_4 , m_{13}}

It is obvious that space contains a table of lines and plans and a table of planes now we will create a table for the planes and planes that will be obtained according to the following equation: $a_1b_1+a_2b_2+a_3b_3+a_4b_4=0$when $m_i=[a_1,a_2,a_3,a_4]$ and $m_j=[b_1,b_2,b_3,b_4]$

Table 1:Points of planes in PG (3, 2).

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T_{15}	T_{14}	T_{13}	T_{12}	T_{11}	T_{10}	T_9	T_8	T_7	T_6	T_5	T_4	T_3	T_2	T_1
2	2	5	1	4	1	2	3	1	1	3	1	1	1	2
3	7	6	6	5	3	4	5	2	4	4	2	2	3	3
6	8	7	7	6	8	9	10	5	6	5	3	4	4	4
8	9	9	8	8	9	10	11	7	11	7	5	5	7	6
13	11	10	10	9	10	11	12	12	12	8	6	8	9	7
14	12	13	11	12	12	13	14	13	13	11	9	10	14	10
15	15	15	14	14	13	14	15	14	15	13	11	15	15	12

m_i is points and T_i is planes

2. intersection of planes

We take the first plane of Table No. 1 that intersects with the planes and denoted by the result of the intersection as S_i $i = 1, \ldots, 105$, for example $S_1 = T_1 \cap T_2$, $S_3 = T_1 \cap T_4$, $S_5 = T_1 \cap T_6$, $S_7 = T_1 \cap T_8$, $S_9 = T_1 \cap T_{10}$, $S_{11} = T_1 \cap T_{12}$, $S_{13} = T_1 \cap T_{14}$, and so on for the rest of the planes. Through the intersection, we obtain 105 projective lines each containing three points ,as shown in Table No. 2.

Table 2:intersection of planes

Points of intersection planes	S_i	Points of intersection planes	S_i
{2,4,10}	$S_2 = T_1 \cap T_3$	{3,4,7}	$S_1 = T_1 \cap T_2$
{3,4,7}	$S_4 = T_1 \cap T_5$	{2,3,6}	$S_3 = T_1 \cap T_4$
{2,7,12}	$S_6 = T_1 \cap T_7$	{4,6,12}	$S_5 = T_1 \cap T_6$
{2,4,10}	$S_8 = T_1 \cap T_9$	{3,10,12}	$S_7 = T_1 \cap T_8$

{4,6,12}	$S_{10} = T_1 \cap T_{11}$	{3,10,12}	$S_9 = T_1 \cap T_{10}$
{6,7,10}	$S_{12} = T_1 \cap T_{13}$	{6,7,10}	$S_{11} = T_1 \cap T_{12}$
{2,3,6}	$S_{14} = T_1 \cap T_{15}$	{2,7,12}	$S_{13} = T_1 \cap T_{14}$
{1,3,9}	$S_{16} = T_2 \cap T_4$	{1,4,15}	$S_{15} = T_2 \cap T_3$
{1,4,15}	$S_{18} = T_2 \cap T_6$	{3,4,7}	$S_{17} = T_2 \cap T_5$
{3,14,15}	$S_{20} = T_2 \cap T_8$	{1,7,14}	$S_{19} = T_2 \cap T_7$
{1,3,9}	$S_{22} = T_2 \cap T_{10}$	{4,9,14}	$S_{21} = T_2 \cap T_9$
{1,7,14}	$S_{24} = T_2 \cap T_{12}$	{4,9,14}	$S_{23} = T_2 \cap T_{11}$
{7,9,15}	$S_{26} = T_2 \cap T_{14}$	{7,9,15}	$S_{25} = T_2 \cap T_{13}$
{1,2,5}	$S_{28} = T_3 \cap T_4$	{3,14,15}	$S_{27} = T_2 \cap T_{15}$
{1,4,15}	$S_{30} = T_3 \cap T_6$	{4,5,8}	$S_{29} = T_3 \cap T_5$
{5,10,15}	$S_{32} = T_3 \cap T_8$	{1,2,5}	$S_{31} = T_3 \cap T_7$
{1,8,10}	$S_{34} = T_3 \cap T_{10}$	{2,4,10}	$S_{33} = T_3 \cap T_9$
{1,8,10}	$S_{36} = T_3 \cap T_{12}$	{4,5,8}	$S_{35} = T_3 \cap T_{11}$
{2,8,15}	$S_{38} = T_3 \cap T_{14}$	{5,10,15}	$S_{37} = T_3 \cap T_{13}$
{3,5,11}	$S_{40} = T_4 \cap T_5$	{2,8,15}	$S_{39} = T_3 \cap T_{15}$
{1,2,5}	$S_{42} = T_4 \cap T_7$	{1,6,11}	$S_{41} = T_4 \cap T_6$
{2,9,11}	$S_{44} = T_4 \cap T_9$	{3,5,11}	$S_{43} = T_4 \cap T_8$
{5,6,9}	$S_{46} = T_4 \cap T_{11}$	{1,3,9}	$S_{45} = T_4 \cap T_{10}$
{5,6,9}	$S_{48} = T_4 \cap T_{13}$	{1,6,11}	$S_{47} = T_4 \cap T_{12}$
{2,3,6}	$S_{50} = T_4 \cap T_{15}$	{2,9,11}	$S_{49} = T_4 \cap T_{14}$
{5,7,13}	$S_{52} = T_5 \cap T_7$	{4,11,13}	$S_{51} = T_5 \cap T_6$
{4,11,13}	$S_{54} = T_5 \cap T_9$	{3,5,11}	$S_{53} = T_5 \cap T_8$

$\{4,5,8\}$	$S_{56} = T_5 \cap T_{11}$	$\{3,8,13\}$	$S_{55} = T_5 \cap T_{10}$
$\{5,7,13\}$	$S_{58} = T_5 \cap T_{13}$	$\{7,8,11\}$	$S_{57} = T_5 \cap T_{12}$
$\{3,8,13\}$	$S_{60} = T_5 \cap T_{15}$	$\{7,8,11\}$	$S_{59} = T_5 \cap T_{14}$
$\{11,12,15\}$	$S_{62} = T_6 \cap T_8$	$\{1,12,13\}$	$S_{61} = T_6 \cap T_7$
$\{1,12,13\}$	$S_{64} = T_6 \cap T_{10}$	$\{4,11,13\}$	$S_{63} = T_6 \cap T_9$
$\{1,6,11\}$	$S_{66} = T_6 \cap T_{12}$	$\{4,6,12\}$	$S_{65} = T_6 \cap T_{11}$
$\{11,12,15\}$	$S_{68} = T_6 \cap T_{14}$	$\{6,13,15\}$	$S_{67} = T_6 \cap T_{13}$
$\{5,12,14\}$	$S_{70} = T_7 \cap T_8$	$\{6,13,15\}$	$S_{69} = T_6 \cap T_{15}$
$\{1,12,13\}$	$S_{72} = T_7 \cap T_{10}$	$\{2,13,14\}$	$S_{71} = T_7 \cap T_9$
$\{1,7,14\}$	$S_{74} = T_7 \cap T_{12}$	$\{5,12,14\}$	$S_{73} = T_7 \cap T_{11}$
$\{2,7,12\}$	$S_{76} = T_7 \cap T_{14}$	$\{5,7,13\}$	$S_{75} = T_7 \cap T_{13}$
$\{10,11,14\}$	$S_{78} = T_8 \cap T_9$	$\{2,13,14\}$	$S_{77} = T_7 \cap T_{15}$
$\{5,12,14\}$	$S_{80} = T_8 \cap T_{11}$	$\{3,10,12\}$	$S_{79} = T_8 \cap T_{10}$
$\{5,10,15\}$	$S_{82} = T_8 \cap T_{13}$	$\{10,11,14\}$	$S_{81} = T_8 \cap T_{12}$
$\{3,14,15\}$	$S_{84} = T_8 \cap T_{15}$	$\{11,12,15\}$	$S_{83} = T_8 \cap T_{14}$
$\{4,9,14\}$	$S_{86} = T_9 \cap T_{11}$	$\{9,10,13\}$	$S_{85} = T_9 \cap T_{10}$
$\{9,1013\}$	$S_{88} = T_9 \cap T_{13}$	$\{10,11,14\}$	$S_{87} = T_9 \cap T_{12}$
$\{2,13,14\}$	$S_{90} = T_9 \cap T_{15}$	$\{2,9,11\}$	$S_{89} = T_9 \cap T_{14}$
$\{1,8,10\}$	$S_{92} = T_{10} \cap T_{12}$	$\{8,9,12\}$	$S_{91} = T_{10} \cap T_{11}$
$\{8,9,12\}$	$S_{94} = T_{10} \cap T_{14}$	$\{9,10,13\}$	$S_{93} = T_{10} \cap T_{13}$
$\{6,8,14\}$	$S_{96} = T_{11} \cap T_{12}$	$\{3,8,13\}$	$S_{95} = T_{10} \cap T_{15}$
$\{8,9,12\}$	$S_{98} = T_{11} \cap T_{14}$	$\{5,6,9\}$	$S_{97} = T_{11} \cap T_{13}$
$\{6,7,10\}$	$S_{100} = T_{12} \cap T_{13}$	$\{6,8,14\}$	$S_{99} = T_{11} \cap T_{15}$

$\{6,8,14\}$	$S_{102} = T_{12} \cap T_{15}$	$\{7,8,11\}$	$S_{101} = T_{12} \cap T_{14}$
$\{6,13,15\}$	$S_{104} = T_{13} \cap T_{15}$	$\{7,9,15\}$	$S_{103} = T_{13} \cap T_{14}$
		$\{2,8,15\}$	$S_{105} = T_{14} \cap T_{15}$

3. S_i of union of the with Frame

We take the S_1 of Table No. 2 that union S_i with the frame , $F = \{1,2,3,4,13\}$and denoted by the result of the union as C_i $i = 1, \dots ,105$, for example $C_1 = S_1 \cup F = \{1,2,3,4,7,13\}$ as shown in Table No. 3.

Table 3: S_i **of union of the with Frame.**

C_i $i = 2,4,6, \dots ,104$		$S_i \cup F$ $i = 2,4,6, \dots ,104$	C_i $i = 1,3,5, \dots ,105$		$S_i \cup F$ $i = 1,3,5, \dots ,105$
$\{1,2,3,4,10,13\}$	C_2	$S_2 \cup F$	$\{1,2,3,4,7,13\}$	C_1	$S_1 \cup F$
$\{1,2,3,4,7,13\}$	C_4	$S_4 \cup F$	$\{1,2,3,4,6,13\}$	C_3	$S_3 \cup F$
$\{1,2,3,4,7,12,13\}$	C_6	$S_6 \cup F$	$\{1,2,3,4,6,12,13\}$	C_5	$S_5 \cup F$
$\{1,2,3,4,7,13\}$	C_8	$S_8 \cup F$	$\{1,2,3,4,10,12,13\}$	C_7	$S_7 \cup F$
$\{1,2,3,4,6,12,13\}$	C_{10}	$S_{10} \cup F$	$\{1,2,3,4,10,12,13\}$	C_9	$S_9 \cup F$
$\{1,2,3,4,6,7,12,13\}$	C_{12}	$S_{12} \cup F$	$\{1,2,3,4,6,7,10,13\}$	C_{11}	$S_{11} \cup F$
$\{1,2,3,4,6,13\}$	C_{14}	$S_{13} \cup F$	$\{1,2,3,4,7,12,13\}$	C_{13}	$S_{12} \cup F$
$\{1,2,3,4,9,13\}$	C_{16}	$S_{16} \cup F$	$\{1,2,3,4,13,15\}$	C_{15}	$S_{15} \cup F$
$\{1,2,3,4,13,15\}$	C_{18}	$S_{18} \cup F$	$\{1,,2,3,4,7,13\}$	C_{17}	$S_{17} \cup F$
$\{1,2,3,4,13,14,15\}$	C_{20}	$S_{20} \cup F$	$\{1,2,3,4,7,13,14\}$	C_{19}	$S_{19} \cup F$

$\{1,2,3,4,9,13\}$	C_{22}	$S_{22} \cup F$	$\{1,2,3,4,9,13,14\}$	C_{21}	$S_{21} \cup F$
$\{1,2,3,4,7,13,14\}$	C_{24}	$S_{24} \cup F$	$\{1,2,3,4,9,13,14\}$	C_{23}	$S_{23} \cup F$
$\{1,2,3,4,7,9,13,15\}$	C_{26}	$S_{26} \cup F$	$\{1,2,3,4,7,9,13,15\}$	C_{25}	$S_{25} \cup F$
$\{1,2,3,4,5,13\}$	C_{28}	$S_{28} \cup F$	$\{1,2,3,4,13,14,15\}$	C_{27}	$S_{27} \cup F$
$\{1,2,3,4,13,15\}$	C_{30}	$S_{30} \cup F$	$\{1,2,3,4,5,8,13\}$	C_{29}	$S_{29} \cup F$
$\{1,2,3,4,5,10,13,15\}$	C_{32}	$S_{32} \cup F$	$\{1,2,3,4,5,13\}$	C_{31}	$S_{31} \cup F$
$\{1,2,3,4,8,10,13\}$	C_{34}	$S_{34} \cup F$	$\{1,2,3,4,10,13\}$	C_{33}	$S_{33} \cup F$
$\{1,2,3,4,8,10,13\}$	C_{36}	$S_{36} \cup F$	$\{1,2,3,4,5,8,13\}$	C_{35}	$S_{35} \cup F$
$\{1,2,3,4,8,13,15\}$	C_{38}	$S_{38} \cup F$	$\{1,2,3,4,5,10,13,15\}$	C_{37}	$S_{37} \cup F$
$\{1,2,3,4,5,11,13\}$	C_{40}	$S_{40} \cup F$	$\{1,2,3,4,8,13,15\}$	C_{39}	$S_{39} \cup F$
$\{1,2,3,4,5,13\}$	C_{42}	$S_{42} \cup F$	$\{1,2,3,4,6,11,13\}$	C_{41}	$S_{41} \cup F$
$\{1,2,3,4,9,11,13\}$	C_{44}	$S_{44} \cup F$	$\{1,2,3,4,5,11,13\}$	C_{43}	$S_{43} \cup F$
$\{1,2,3,4,5,6,9,13\}$	C_{46}	$S_{46} \cup F$	$\{1,2,3,4,9,13\}$	C_{45}	$S_{45} \cup F$
$\{1,2,3,4,5,6,9,13\}$	C_{48}	$S_{48} \cup F$	$\{1,2,3,4,6,11,13\}$	C_{47}	$S_{47} \cup F$
$\{1,2,3,4,6,13\}$	C_{50}	$S_{50} \cup F$	$\{1,2,3,4,9,11,13\}$	C_{49}	$S_{49} \cup F$
$\{1,2,3,4,5,7,13\}$	C_{52}	$S_{52} \cup F$	$\{1,2,3,4,11,13\}$	C_{51}	$S_{51} \cup F$
$\{1,2,3,4,11,13\}$	C_{54}	$S_{54} \cup F$	$\{1,2,3,4,5,11,13\}$	C_{53}	$S_{53} \cup F$
$\{1,2,3,4,5,8,13\}$	C_{56}	$S_{56} \cup F$	$\{1,2,3,4,8,13\}$	C_{55}	$S_{55} \cup F$
$\{1,2,3,4,5,7,13\}$	C_{58}	$S_{58} \cup F$	$\{1,2,3,4,7,8,11,13\}$	C_{57}	$S_{57} \cup F$
$\{1,2,3,4,8,13\}$	C_{60}	$S_{60} \cup F$	$\{1,2,3,4,7,8,11,13\}$	C_{59}	$S_{59} \cup F$
$\{1,2,3,4,11,12,13,$	C_{62}	$S_{62} \cup F$	$\{1,2,3,4,12,13\}$	C_{61}	$S_{61} \cup F$

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$15\}$					
$\{1,2,3,4,12,13\}$	C_{64}	$S_{64} \cup F$	$\{1,2,3,4,11,13\}$	C_{63}	$S_{63} \cup F$
$\{1,2,3,4,6,11,13\}$	C_{66}	$S_{66} \cup F$	$\{1,2,3,4,6,12,13\}$	C_{65}	$S_{65} \cup F$
$\{1,2,3,4,11,12,13,15\}$	C_{68}	$S_{68} \cup F$	$\{1,2,3,4,6,13,15\}$	C_{67}	$S_{67} \cup F$
$\{1,2,3,4,5,12,13,14\}$	C_{70}	$S_{70} \cup F$	$\{1,2,3,4,6,13,15\}$	C_{69}	$S_{69} \cup F$
$\{1,2,3,4,12,13\}$	C_{72}	$S_{72} \cup F$	$\{1,2,3,4,13,14\}$	C_{71}	$S_{71} \cup F$
$\{1,2,3,4,7,13,14\}$	C_{74}	$S_{74} \cup F$	$\{1,2,3,4,5,12,13,14\}$	C_{73}	$S_{73} \cup F$
$\{1,2,3,4,7,12,13\}$	C_{76}	$S_{76} \cup F$	$\{1,2,3,4,5,7,13\}$	C_{75}	$S_{75} \cup F$
$\{1,2,3,4,10,11,13,14\}$	C_{78}	$S_{78} \cup F$	$\{1,2,3,4,13,14]$	C_{77}	$S_{77} \cup F$
$\{1,2,3,4,5,12,13,14\}$	C_{80}	$S_{80} \cup F$	$\{1,2,3,4,10,12,13\}$	C_{79}	$S_{79} \cup F$
$\{1,2,3,4,5,10,13,15\}$	C_{82}	$S_{82} \cup F$	$\{1,2,3,4,10,11,13,14\}$	C_{81}	$S_{81} \cup F$
$\{1,2,3,4,13,14,15\}$	C_{84}	$S_{84} \cup F$	$\{1,2,3,4,11,12,13,15\}$	C_{83}	$S_{83} \cup F$
$\{1,2,3,4,9,13,14\}$	C_{86}	$S_{86} \cup F$	$\{1,2,3,4,9,10,13\}$	C_{85}	$S_{85} \cup F$
$\{1,2,3,4,9,10,13\}$	C_{88}	$S_{88} \cup F$	$\{1,2,3,4,10,11,13,14\}$	C_{87}	$S_{87} \cup F$
$\{1,2,3,4,13,14\}$	C_{90}	$S_{90} \cup F$	$\{1,2,3,4,9,11,13\}$	C_{89}	$S_{89} \cup F$
$\{1,2,3,4,8,10,13\}$	C_{92}	$S_{92} \cup F$	$\{1,2,3,4,8,9,12,13\}$	C_{91}	$S_{91} \cup F$
$\{1,2,3,4,8,9,12,13\}$	C_{94}	$S_{94} \cup F$	$\{1,2,3,4,9,10,13\}$	C_{93}	$S_{93} \cup F$

{1,2,3,4,6,8,13,14}	C_{96}	$S_{96} \cup F$	{1,2,3,4,8,13}	C_{95}	$S_{95} \cup F$
{1,2,3,4,8,9,12,13}	C_{98}	$S_{98} \cup F$	{1,2,3,4,5,6,9,13}	C_{97}	$S_{97} \cup F$
{1,2,3,4,6,7,10,13}	C_{100}	$S_{100} \cup F$	{1,2,3,4,6,8,13,14}	C_{99}	$S_{99} \cup F$
{1,2,3,4,6,8,13,14}	C_{102}	$S_{102} \cup F$	{1,2,3,4,7,8,11,13}	C_{101}	$S_{101} \cup F$
{1,2,3,4,6,13,15}	C_{104}	$S_{104} \cup F$	{1,2,3,4,7,9,13,15}	C_{103}	$S_{103} \cup F$
			{1,2,3,4,8,13,15}	C_{105}	$S_{105} \cup F$

4. Linear codes of C_i ,i = 1,2, ... ,105

We take C_1 and put it in the form of a matrix. We notice that it is the generator matrix G and we calculate x , n, and d through the matrix, for example $C_1 = \{1,2,3,4,7,13\}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

x = 4, n = 6, y = 2

as shown in Table No. 4.

Table 4: Linear codes of C_i

Generator matrix G	$[n,x,y]_2 - code$	C_i	Generator matrix G	$[n,x,y]_2 - code$	C_i
$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$	$[6,4,2]_2 - code$	C_2	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$	$[6,4,2]_2 - code$	C_1

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$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_4	$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_3
$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&0&1&1\\0&0&1&0&1&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_6	$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_5
$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_8	$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_7
$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{10}	$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_9
$\begin{bmatrix}1&0&0&0&0&0&0&1\\0&1&0&0&1&0&1&1\\0&0&1&0&1&1&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,2]_2$ $-code$	C_{12}	$\begin{bmatrix}1&0&0&0&0&0&0&1\\0&1&0&0&1&0&1&1\\0&0&1&0&1&1&0&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,2]_2$ $-code$	C_{11}
$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{14}	$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&0&1\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{13}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{16}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&0\\0&0&1&0&1&0\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{15}

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$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&0\\0&0&1&0&1&0\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{18}	$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{17}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{20}	$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&0&1&0\\0&0&1&0&1&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{19}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{22}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&1&0\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{21}
$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&0&1&0\\0&0&1&0&1&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{24}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&1&0\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{23}
$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&0&0&1&0\\0&0&1&0&1&1&1&0\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,2]_2$ $-code$	C_{26}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&0&0&1&0\\0&0&1&0&1&1&1&0\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,2]_2$ $-code$	C_{25}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{28}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&0&0\\0&0&1&0&1&1&0\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{27}

$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&0\\0&0&1&0&1&0\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{30}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&0&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{29}
$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&0&1&0\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,2]_2-code$	C_{32}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{31}
$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&1&1&1\\0&0&1&0&0&0&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{34}	$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{33}
$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&1&1&1\\0&0&1&0&0&0&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{36}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&0&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{35}
$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&0&1&0\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{38}	$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&0&1&0\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,2]_2-code$	C_{37}
$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{40}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&0&1&0\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{39}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{42}	$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{41}

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$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{44}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{43}
$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&0&1\\0&0&1&0&0&1&1&1\\0&0&0&1&0&0&0&1\end{bmatrix}$	$[8,4,2]_2$ $-code$	C_{46}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{45}
$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&0&1\\0&0&1&0&0&1&1&1\\0&0&0&1&0&0&0&1\end{bmatrix}$	$[8,4,2]_2$ $-code$	C_{48}	$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{47}
$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{50}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{49}
$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&1&0&1\\0&0&1&0&0&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2$ $-code$	C_{52}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{51}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2$ $-code$	C_{54}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2$ $-code$	C_{53}

$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&1\\0&0&1&0&0&0&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{56}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2 - code$	C_{55}
$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&1&0&1\\0&0&1&0&0&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2 - code$	C_{58}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&0&1&1&1\\0&0&1&0&1&0&1&1\\0&0&0&1&1&1&0&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{57}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2 - code$	C_{60}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&0&1&1&1\\0&0&1&0&1&0&1&1\\0&0&0&1&1&1&0&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{59}
$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&1&1&1&0\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{62}	$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2 - code$	C_{61}
$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2 - code$	C_{64}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&0&1\end{bmatrix}$	$[6,4,2]_2 - code$	C_{63}
$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{66}	$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{65}

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$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&1&1&1&0\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{68}	$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&1&0\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2-code$	C_{67}
$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&1&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{70}	$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&1&0\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2-code$	C_{69}
$\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&1\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{72}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&0\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{71}
$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&0&1&0\\0&0&1&0&1&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{74}	$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&1&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{73}
$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&0&1&1\\0&0&1&0&1&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{76}	$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&1&0&1\\0&0&1&0&0&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2-code$	C_{75}
$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&1&1&1\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{78}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&0\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{77}
$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&1&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{80}	$\begin{bmatrix}1&0&0&0&0&0&1\\0&1&0&0&1&1&1\\0&0&1&0&0&1&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{79}

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$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&0&1&0\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,2]_2 - code$	C_{82}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&1&1&1\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{81}
$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&0&0\\0&0&1&0&1&1&0\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{84}	$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&1&1&1&0\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{83}
$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&1&0\\0&0&1&0&1&1&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{86}	$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&0&1&1\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2 - code$	C_{85}
$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&0&1&1\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2 - code$	C_{88}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&0&1&1&1\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{87}
$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&0\\0&0&1&0&1&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2 - code$	C_{90}	$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&0&1&1\\0&0&1&0&1&1&1\\0&0&0&1&0&0&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{89}
$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&1&1&1\\0&0&1&0&0&0&1\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2 - code$	C_{92}	$\begin{bmatrix}1&0&0&0&1&1&0&1\\0&1&0&0&1&0&1&1\\0&0&1&0&0&1&1&1\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,4]_2 - code$	C_{91}

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$\begin{bmatrix}1&0&0&0&1&1&0&1\\0&1&0&0&1&0&1&1\\0&0&1&0&0&1&1&1\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{94}	$\begin{bmatrix}1&0&0&0&1&0&1\\0&1&0&0&0&1&1\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2-code$	C_{93}
$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&1&0&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{96}	$\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&1&1\\0&0&1&0&0&1\\0&0&0&1&1&1\end{bmatrix}$	$[6,4,2]_2-code$	C_{95}
$\begin{bmatrix}1&0&0&0&1&1&0&1\\0&1&0&0&1&0&1&1\\0&0&1&0&0&1&1&1\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{98}	$\begin{bmatrix}1&0&0&0&1&0&1&1\\0&1&0&0&1&1&0&1\\0&0&1&0&0&1&1&1\\0&0&0&1&0&0&0&1\end{bmatrix}$	$[8,4,2]_2-code$	C_{97}
$\begin{bmatrix}1&0&0&0&0&0&0&1\\0&1&0&0&1&0&1&1\\0&0&1&0&1&1&0&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,2]_2-code$	C_{100}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&1&0&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{99}
$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&1&1&1&0\\0&0&1&0&1&0&1&1\\0&0&0&1&0&1&1&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{102}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&0&1&1&1\\0&0&1&0&1&0&1&1\\0&0&0&1&1&1&0&1\end{bmatrix}$	$[8,4,4]_2-code$	C_{101}
$\begin{bmatrix}1&0&0&0&0&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&1&0\\0&0&0&1&0&1&1\end{bmatrix}$	$[7,4,3]_2-code$	C_{104}	$\begin{bmatrix}1&0&0&0&0&1&1&1\\0&1&0&0&0&0&1&0\\0&0&1&0&1&1&1&0\\0&0&0&1&1&0&1&1\end{bmatrix}$	$[8,4,2]_2-code$	C_{103}
			$\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&0&1&0\\0&0&0&1&1&1&1\end{bmatrix}$	$[7,4,2]_2-code$	C_{105}

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5.We start testing the ideality of the codes by applying the (Griesmer bound):

(1) $\{ C_1 , C_2 , C_3 , C_4 , C_{14} , C_{15} , C_{16} , C_{18}, C_{20} , C_{22} , C_{28} , C_{30} , C_{31} , C_{33} , C_{42} , C_{45} , C_{50} , C_{51} , C_{54} , C_{55} , C_{60} , C_{61} , C_{63} , C_{64} , , C_{71} , C_{72} , C_{77} , C_{90} , C_{95} \} = [6,4,2]_2 - code$

$n \geq g_h(x,y) = \sum_{i=0}^{x-1} \left\lceil \frac{y}{h^i} \right\rceil$.

$\sum_{i=0}^{3} \left\lceil \frac{y}{h^i} \right\rceil = \left\lceil \frac{2}{2^0} \right\rceil + \left\lceil \frac{2}{2^1} \right\rceil + \left\lceil \frac{2}{2^2} \right\rceil + \left\lceil \frac{2}{2^3} \right\rceil$

= 4 is optimal linear code

(2) $\{ C_5 , C_6 , C_7 , C_8 , C_9 , C_{10} , C_{13} , C_{17}, C_{19} , C_{21} , C_{23} , C_{24} , C_{27} , C_{29} , C_{34} , C_{35} , C_{36} , C_{38} , C_{39} , C_{40} , C_{41} , C_{43} , C_{44} , C_{47} , , C_{49} , C_{53} , C_{56} , C_{65} , C_{66} , C_{74} , C_{76} , C_{79} , C_{84} , C_{86} , C_{89} , C_{92} , C_{105} \} = [7,4,2]_2 - code$

$$n \geq g_h(x,y) = \sum_{i=0}^{x-1} \left\lceil \frac{y}{h^i} \right\rceil$$

$\sum_{i=0}^{3} \left\lceil \frac{y}{h^i} \right\rceil = \left\lceil \frac{2}{2^0} \right\rceil + \left\lceil \frac{2}{2^1} \right\rceil + \left\lceil \frac{2}{2^2} \right\rceil + \left\lceil \frac{2}{2^3} \right\rceil$

= 4 is optimal linear code

(3) $\{ C_{11} , C_{12} , C_{25} , C_{26} , C_{32} , C_{37} , C_{46} , C_{48}, C_{82} , C_{97} , C_{100} , C_{103} \} = [8,4,2]_2 - code$

$$n \geq g_h(x,y) = \sum_{i=0}^{x-1} \left\lceil \frac{y}{h^i} \right\rceil$$

$\sum_{i=0}^{3} \left\lceil \frac{y}{h^i} \right\rceil = \left\lceil \frac{2}{2^0} \right\rceil + \left\lceil \frac{2}{2^1} \right\rceil + \left\lceil \frac{2}{2^2} \right\rceil + \left\lceil \frac{2}{2^3} \right\rceil$

= 4 is optimal linear code

(4) $\{ C_{57} , C_{59} , C_{62} , C_{68} , C_{70} , C_{73} , C_{78} , C_{80}, C_{81} , C_{83} , C_{87} , C_{91} \; C_{94} , C_{96} , C_{98} , C_{99} , C_{101} , C_{102} \} = [8,4,4]_2 - code$

$$n \geq g_h(x,y) = \sum_{i=0}^{x-1} \left\lceil \frac{y}{h^i} \right\rceil$$

$\sum_{i=0}^{3} \left\lceil \frac{y}{h^i} \right\rceil = \left\lceil \frac{4}{2^0} \right\rceil + \left\lceil \frac{4}{2^1} \right\rceil + \left\lceil \frac{4}{2^2} \right\rceil + \left\lceil \frac{4}{2^3} \right\rceil$

= 8 is optimal linear code

(5) $\{ C_{52}, C_{58}, C_{67}, C_{69}, C_{75}, C_{85}, C_{88}, C_{93}, C_{104} \} = [7,4,3]_2 - code$

$$n \geq g_h(x,y) = \sum_{i=0}^{x-1} \left\lceil \frac{y}{h^i} \right\rceil$$

$\sum_{i=0}^{3} \left\lceil \frac{y}{h^i} \right\rceil = \left\lceil \frac{3}{2^0} \right\rceil + \left\lceil \frac{3}{2^1} \right\rceil + \left\lceil \frac{3}{2^2} \right\rceil + \left\lceil \frac{3}{2^3} \right\rceil$

= 6 is optimal linear code

Result

	Group	$[n,x,y]_h - code$	Griesmer bound	Status
1	$\{ C_1, C_2, C_3, C_4, C_{14}, C_{15}, C_{16}, C_{18}, C_{20}, C_{22}, C_{28}, C_{30}, C_{31}, C_{33}, C_{42}, C_{45}, C_{50}, C_{51}, C_{54}, C_{55}, C_{60}, C_{61}, C_{63}, C_{64}, , C_{71}, C_{72}, C_{77}, C_{90}, C_{95} \}$	$[6,4,2]_2 - code$	Verified	Optimal linear code
2	$\{ C_5, C_6, C_7, C_8, C_9, C_{10}, C_{13}, C_{17}, C_{19}, C_{21}, C_{23}, C_{24}, C_{27}, C_{29}, C_{34}, C_{35}, C_{36}, C_{38}, C_{39}, C_{40}, C_{41}, C_{43}, C_{44}, C_{47}, , C_{49}, C_{53}, C_{56}, C_{65}, C_{66}, C_{74}, C_{76}, C_{79}, C_{84}, C_{86}, C_{89}, C_{92}, C_{105} \}$	$[7,4,2]_2 - code$	Verified	Optimal linear code
3	$\{ C_{57}, C_{59}, C_{62}, C_{68}, C_{70}$	$[8,4,4]_2 - code$	Verified	Optimal

	, C_{73} , C_{78} , C_{80}, C_{81} , C_{83} , C_{87} , C_{91} C_{94} , C_{96} , C_{98} , C_{99} , C_{101} , C_{102} }			linear code
4	{ C_{11} , C_{12} , C_{25} , C_{26} , C_{32} , C_{37} , C_{46} , C_{48}, C_{82} , C_{97} , C_{100} , C_{103}}	$[8,4,2]_2 - code$	Verified	Optimal linear code
5	{ C_{52} , C_{58} , C_{67} , C_{69} , C_{75} , C_{85} , C_{88} , C_{93}, C_{104} }	$[7,4,3]_2 - code$	Verified	Optimal linear code

Conclusion

Based on a study of plane intersections in projective space defined over the binary field (field 2), 105 linear codes were constructed and classified into five groups according to their parameters $[n,x,y]_h - code$:

Group 1: 29 with parameters$[6,4,2]_2 - code$

Group 2: 37 has parameters $[7,4,2]_2 - code$

Group 3: 18 has $[8,4,4]_2 - code$

Group 4: 12 has $[8,4,2]_2 - code$

Group 5: 9 has $[7,4,3]_2 - code$

Furthermore, a precise application of the Griesmer bound demonstrated that all these resulting codes are optimal linear codes.

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